

Admissibles and computations

Bruno Durand Grégory Lafitte

LIRMM – CNRS – Montpellier

GT Calculabilités – Nov 25-27, 2019



LIRMM



*Quelqu'un va-t-il prendre enfin la défense
de l'infini ?
—Aragon, Paris-Journal, 13 avril 1923*

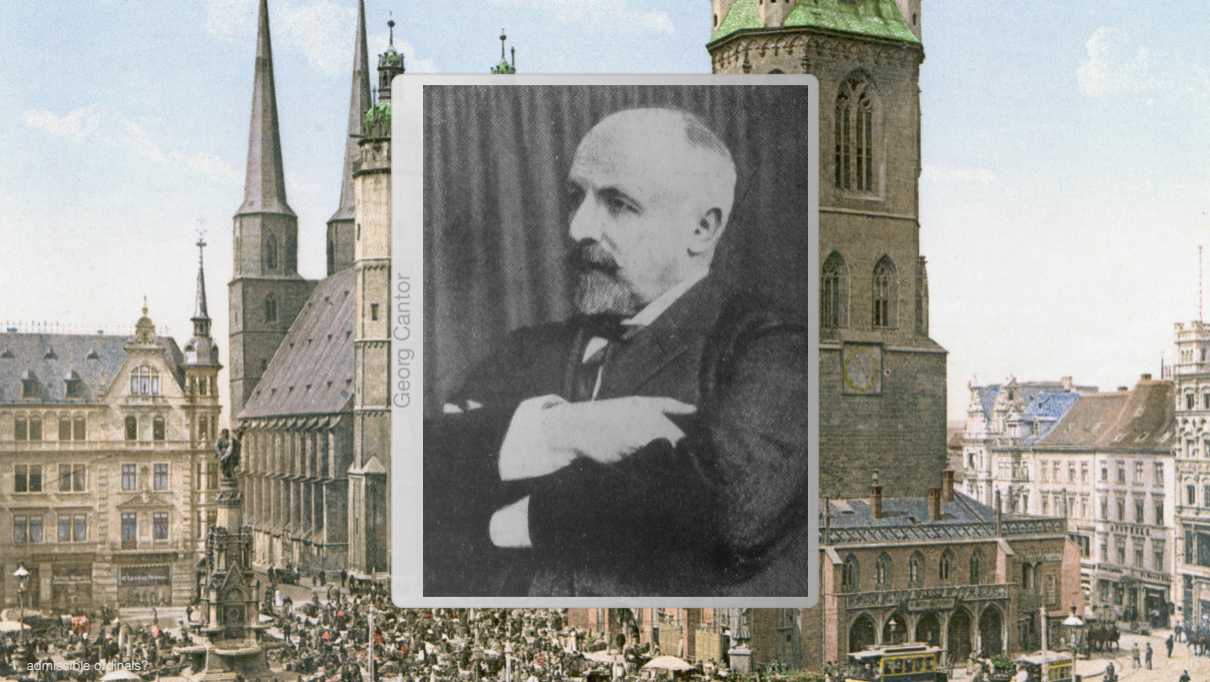
AGENCE NATIONALE DE LA RECHERCHE
ANR

1. admissible ordinals?

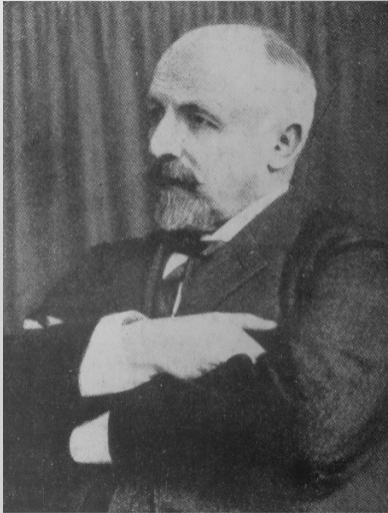
Cantor a trouvé une loi d'engendrement de la multitude des nombres ordinaux finis et transfinis, il a trouvé une dynastie, celle des Aleph, et cela, à l'aide de deux principes seulement, l'un immanent (additif), l'autre transcendant (passage à la limite) : Cantor, législateur de l'infini.

—Sinisgalli, Horror vacui





Georg Cantor



Ordinals, a whole bunch of them

DEF

Equivalence class of well-orderings by isomorphism

Ordinals, a whole bunch of them

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Equivalence class of well-orderings by isomorphism

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Transitive set well-ordered by \in



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Ordinals + transfinite induction

measuring of provability strength

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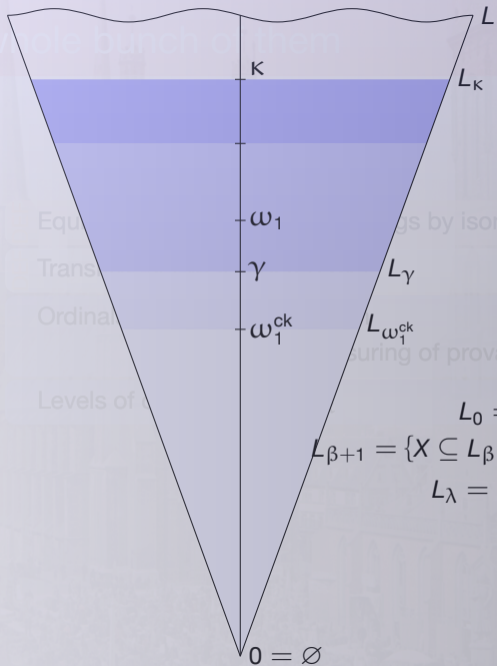
Ordinals + transfinite induction

measuring of provability strength

Levels of constructibility L_α

Ordinals, a whole bunch of them

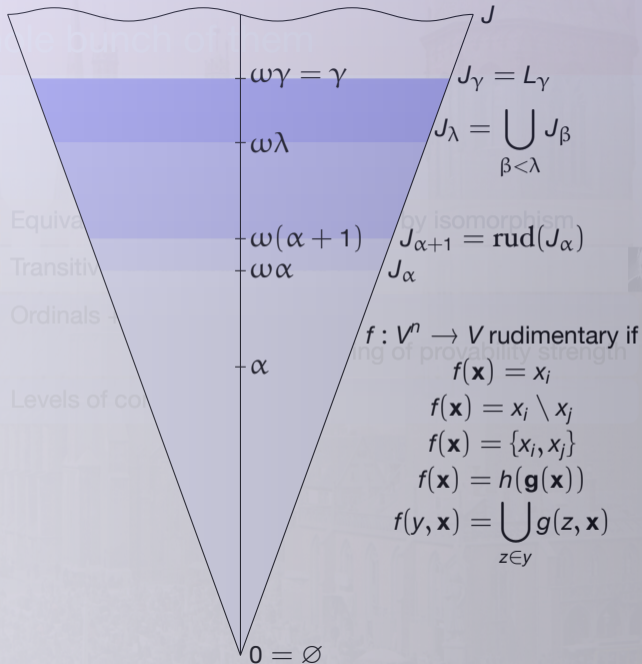
Constructible Universe



$$L_0 = \emptyset$$
$$L_{\beta+1} = \{X \subseteq L_\beta \text{ def.}\} = \mathbf{Def}(L_\beta)$$
$$L_\lambda = \bigcup_{\beta < \lambda} L_\beta$$

Ordinals, a whole bunch of them

Constructible Universe



Equiva

Transitiv

Ordinals

Levels of co



by isomorphism

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Coding infinite ordinals

Ordinals, a whole bunch of them

$$\langle x, y \rangle = 1 \text{ iff } x < y$$

$\langle \langle \cdot, \cdot \rangle$ codage de \mathbb{N}^2 dans \mathbb{N})

ordre sur ω

coding

ordre sur ω vs ordre sur une partie de ω

(soit n est comparable à une infinité d'entiers, soit n est isolé)

si $\forall x \langle n, x \rangle = 0$, alors soit n est le plus petit élément, soit il n'est pas dans l'ordre.

si de plus $\forall x \langle x, n \rangle = 0$ alors x n'est pas dans l'ordre

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An ordinal is recursive if it has a recursive coding on ω

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If α has a recursive coding on $E \subset \omega$ then it is recursive

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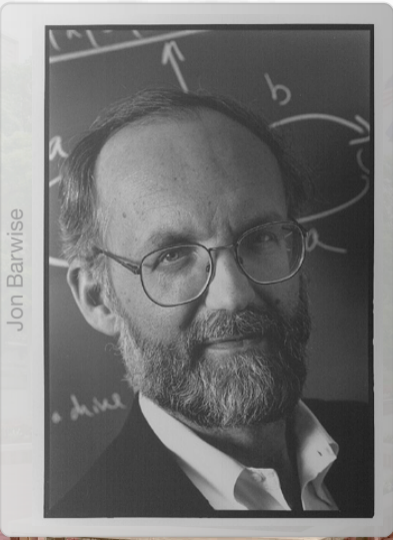
A code for $\beta < \alpha$ extracted from a code for α



Indiana University



Jon Barwise



Admissible ordinals

closed enough ordinals

limit, limit of limits, etc

Admissible ordinals

closed enough ordinals

limit, limit of limits, etc

DEEP ω_1^{CK} is the sup of the recursive ordinals

Admissible ordinals

Σ_0 -separation:

for any set E , Σ_0 formula ϕ ,
there exists $X \subseteq E$ such that

$$x \in X \iff \phi(x)$$

Σ_0 -collection:

for any Σ_0 formula $\phi(x, y)$ s.t.

$\forall x \exists y \phi(x, y)$, we have that $\forall x \exists e$
s.t. $[e \in E \iff \exists x \in X \phi(x, e)]$

closed enough ordinals

limit, limit of limits, etc

ω_1^{CK} is the sup of the recursive ordinals

α is *admissible* if L_α is a model of KP

KP

extensionality
induction
empty set
pairing
union
 Σ_0 -separation
 Σ_0 -collection



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α is admissible if limit and $L_\alpha \models \Sigma_0$ -collection

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unreachable in a Σ_1 way from below

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unreachable in a Σ_1 way from below

THM $\omega_1^{\text{CK}, r}$ is admissible for every real r

KP {
extensionality
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2. gaps?

*ITTM*s, *clockable ordinals*, *gaps*.

ITTM: clockable ordinals and gaps

Infinite time Turing machines (ITTM)

DEF

\limsup

special limit state

head limit behaviour

can compute on *reals*



ITTM: clockable ordinals and gaps

ITTMs are extensions of Turing machines to transfinite time.

For simplicity: alphabet $\{0, 1\}$

Ordinal stages

- *successor* ordinals: it works exactly as a Turing machine
- *limit* ordinals: each cell is set to the \limsup of its values, head is rewinded back to the origin and the machine enters a special limit state L

3 tapes: input, scratch, output

A real (infinite binary string) can be considered as input (*oracle* computation) and output

Example: a coding of an ordinal may be written, or taken as input by an ITTM

ITTM and what not

Infinite time Turing machines (ITTM)

\limsup

special limit state

head limit behaviour

can compute on reals



ITTM: clockable ordinals and gaps

Infinite time Turing machines (ITTM)

DEF \limsup special limit state head limit behaviour
can compute on reals

DEF α countable is *writable* if r_α is



ITTM: clockable ordinals and gaps

Infinite time Turing machines (ITTM)



Any recursive ordinal α can be finitely represented by a Turing Machine μ such that $\mu(i) = r_\alpha(i)$

Among these TM we choose one (e.g. of smallest index) $\alpha \mapsto \mu_\alpha$

Our order (partial) : $\mu_\alpha \prec \mu_\beta \iff \alpha < \beta$

This order \prec is not recursive and is of type ω_1^{CK}

This order does not exhibit a recursive minimality — we will improve it using ITTMs

coding ω_1^{CK}

ITTM: clockable ordinals and gaps

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\limsup special limit state head limit behaviour
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DEF

α countable is *writable* if r_α is

α countable is *clockable* if \exists ITTM

which halts exactly after α many steps

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DEF

$\lambda_\infty = \text{sup of writables}$ $\gamma_\infty = \text{sup of clockables}$

ITTM: clockable ordinals and gaps

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DEF $\lambda_\infty = \text{sup of writables}$ $\gamma_\infty = \text{sup of clockables}$

DEF eventually writable ordinals $\zeta_\infty = \text{sup}$
accidentally writable ordinals $\Sigma_\infty = \text{sup}$



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THM There are non-clockable writable ordinals



Clockable ordinals and gaps

THM

Admissibles are not clockable

Clockable ordinals and gaps

THMTHM

Admissibles are not clockable

ω_1^{CK} starts a gap of length ω

Clockable ordinals and gaps

THM THM THM

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A gap length is a limit ordinal

Clockable ordinals and gaps

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$$\gamma_\infty = \lambda_\infty$$

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There are large gaps

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A gap starts by an admissible ordinal

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How are gaps distributed?

Clockable ordinals and gaps

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How are gaps distributed?

Is there a link between a gap's size and its starting point?

Clockable ordinals and gaps

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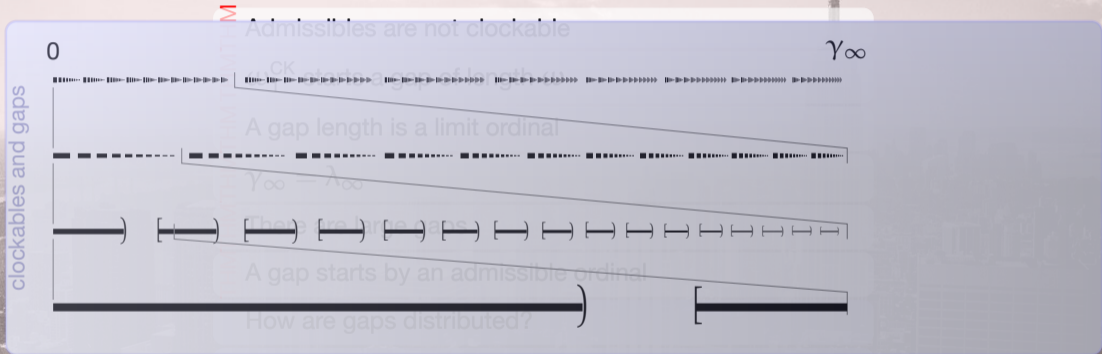
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How are gaps distributed?

Is there a link between a gap's size and its starting point?

Is there a gap with exactly one admissible ordinal properly inside?

Clockable ordinals and gaps



Is there a link between a gap's size and its starting point?

Is there a gap with exactly one admissible ordinal properly inside?

3. a gap with exactly one admissible

One admissible



THM $L_{\lambda_{\infty}} \prec_1 L_{\zeta_{\infty}} \prec_2 L_{\Sigma_{\infty}}$



One admissible



$L_{\lambda_{\infty}} \prec_1 L_{\zeta_{\infty}} \prec_2 L_{\Sigma_{\infty}}$

There is a gap with at least one admissible inside

CORTIM



One admissible

There are many admissible ordinals between the sup of writables λ_∞ and $\zeta_\infty < \Sigma_\infty$.

$\exists \alpha < \beta < \gamma$ s.t. β adm. and L_γ witnesses $[\alpha, \beta)$ contains no clockable

This is a Σ_1 statement, verified in L_{Σ_∞} , and thus also in L_{λ_∞} . Let $\alpha, \beta, \gamma < \lambda_\infty$ witness this. L_γ believes that β is an admissible ordinal properly inside a gap.

Since any ITTM-computation on the empty input of length $< \beta$ is already contained in L_β , $[\alpha, \beta)$ is indeed inside a gap which properly contains the admissible ordinal β .

using $\lambda-\zeta-\lambda$

One admissible

Design an algorithm that checks if a real x is a code for an ordinal which is the starting point of a gap containing $\omega_1^{\text{CK},x}$.

This algorithm accepts any code for λ_∞ .

There is an x that is accepted by this algorithm

is a Σ_1 statement.

By the $\lambda - \zeta - \Sigma$ theorem, we have a witness of this property in L_{λ_∞} , which has to be the code of an admissible ordinal $< \lambda_\infty$ beginning a gap with an admissible properly inside.

algorithmic variant

$L_{\lambda_\infty} \prec_1 L_{\zeta_\infty} \prec_2 L_{\Sigma_\infty}$

there is a gap with at least one admissible inside

One admissible

$$L_{\lambda_{\infty}} \prec_1 L_{\zeta_{\infty}} \prec_2 L_{\Sigma_{\infty}}$$

There is a gap with at least one admissible inside

The first gap with at least an admissible inside will only have one admissible τ , and ends at $\tau + \omega$

THM
CORTHIM



One admissible

Algorithm :

Primary gap detection : we run a universal online-simulation of all ITTM in order to detect gaps. We thus observe gaps ω steps after their starting point.

At each starting points α we have at our disposal a coding of α .

We then start a *secondary gap detection* with oracle α until either :

- the primary gap ends; we then stop secondary detection – we continue the primary detection until next gap
- the first secondary gap is observed; we then halt.

Proof :

If no admissible inside a gap, then it halts after λ_∞ . Contradiction.

Halting time: $\alpha + \omega_1^{\text{CK}, \alpha} + \omega = \omega_1^{\text{CK}, \alpha} + \omega$. Hence this is the end of the gap.

proof

$$\lambda_\infty < \omega_1 < \omega_2 < \omega_3$$

There is a gap with at least one admissible inside

The first gap with at least an admissible inside will only

4. beyond one admissible

Several admissibles, ranks, large gaps

THM The first gap with at least $n < \omega$ admissibles inside will only have n admissibles inside, and ends ω steps after the last one

Several admissibles, ranks, large gaps

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Direct adaptation of previous proof

Gap : $[\alpha_0, \dots, \alpha_1, \dots, \dots, \dots, \alpha_n]$

Computation of ω_1^{CK} relativized to detect the next admissible :

$$\alpha_{i+1} = \omega_1^{\text{CK}, \alpha_i}$$

Several admissibles, ranks, large gaps

THM The first gap with at least $n < \omega$ admissibles inside will only have n admissibles inside, and ends ω steps after the last one

THM ($\alpha < \lambda_\infty$) After the writing time for α , the first gap with at least α admissibles inside has exactly α inside

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Not a direct adaptation of previous proof because of limits of admissibles

Problem: if $\alpha_0, \alpha_1, \dots, \alpha_n, \dots \rightarrow \alpha_\omega$ then sometimes α_ω is admissible (*recursively inaccessible*) and sometimes not.

proof

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DEF Ranks

rank-0: admissibles

rank- $\alpha + 1$: admissible limits of rank α adm.

rank- λ : adm. limits of rank β adm. $\forall \beta < \lambda$

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THM Let $\alpha < \lambda_\infty$. After the writing time for α , the first gap starting with a rank- α admissible is of size ω

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THM Let $f : \omega_1 \rightarrow \omega_1$ be ITTM-computable. Then there is an ordinal α starting a gap of size $\geq f(\alpha)$

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THM ...and a few other improvements

5. sacks' characterizations of *admissible* ordinals

*Il y a en nous une sensation finie de
« l'infini ». Et ce n'est qu'un effet – une con-
séquence. Ce n'est pas une preuve de quoi
que ce soit.* —Paul Valéry, 1910



MCMXVI

MASSACHUSETTS INSTITUTE OF TECHNOLOGY



Gerald E. Sacks



Characterizations of *admissibles*

THM For every countable admissible α , $\exists r$ s.t. $\alpha = \omega_1^{\text{CK},r}$



Characterizations of *admissibles*

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

THM

For every countable admissible α , $\exists r$ s.t. $\alpha = \omega_1^{\text{CK},r}$

How can we understand & prove this in an explicit way?



Characterizations of *admissibles*

simulation of all ITTM's ω -online

ω machines halt for a given simulation time, we choose the first one in the simulation

$\mu \prec \nu \iff (\mu \text{ and } \nu \text{ are chosen and } \mu \text{ halts before } \nu)$

if we run the above process up to α we get an order for all clockables before α

first run of ω non clockables $\implies \omega_1^{\text{CK}}$

the order type of clockables below a gap is exactly the starting point of the gap

coding of λ_∞

same with oracle (or input)

coding of $\omega_1^{\text{CK},r}$

For every countable admissible α , $\exists r$ s.t. $\alpha = \omega_1^{\text{CK},r}$

in some sense these are the *simplest* codings

How can we understand & prove this in an explicit way?

coding ω_1^{CK} and more

Characterizations of *admissibles*

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

THM For every countable admissible α , $\exists r$ s.t. $\alpha = \omega_1^{\text{CK},r}$

How can we understand & prove this in an explicit way?

THM The writing time of any $\alpha < \lambda_\infty$ is the sup of all ends of gaps that start before α



6. building up to a proof

*Qui est là ? Ah très bien : faites entrer
l'infini.*

—Aragon, *Une vague de rêves*, 1924

The first steps



Successor admissible case

The first steps

Successor admissible case

$\alpha = \beta^+$ (α and β are admissible)

Code β in a real r to obtain $\omega_1^{CK,r} = \alpha$

The first steps

Successor admissible case

LEM If r_β codes an ordinal β , $\omega_1^{\text{CK}, r_\beta} \geq \beta^+$

The first steps

Successor admissible case

LEM If r_β codes an ordinal β , $\omega_1^{\text{CK}, r_\beta} \geq \beta^+$

We have to make sure that r does not code more...

The first steps

Successor admissible case

LEM If r_β codes an ordinal β , $\omega_1^{\text{CK}, r_\beta} \geq \beta^+$

We have to make sure that r does not code more...

Find the lowest simplest code

The first steps

Successor admissible case

If coding level of r_β is $< \beta^+$, we have equality and everything works.

Otherwise, we need to do something a little bit more sophisticated.

Find the lowest simplest code

The first steps

Successor admissible case

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Recursively inaccessible case

The first steps

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Recursively inaccessible case

Limit of the ω first admissibles is **not** admissible

The first steps

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We have to make sure that r does not code more...

Find the lowest simplest code

Recursively inaccessible case

Limit of the ω first admissibles is **not** admissible

One can build an ITTM which halts at exactly that limit

The first steps

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Find the lowest simplest code

Recursively inaccessible case

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Consider the first *recursively inaccessible* ι_0



La Sorbonne

The *lost* lemma

We need some tool to construct r

The *lost* lemma

We need some tool to construct r

LEM $\{r_i : i \in \omega\}$ s.t. $\forall i, r_i <_T r_{i+1}$
 $\exists r$ s.t. $\forall i, r_i \leq_T r$ but $\bigoplus_i r_i \not\leq_T r \leq_T (\bigoplus_i r_i)'$

The *lost* lemma

r needs to code the r_i 's but in such a way that $\forall i, r_i$ can be computed from r but not uniformly.

r is a *mapping* from ω^2 to $\{0, 1\}$ that contains r_i in the $\mathfrak{b}(i)$ column.

σχέμα We ensure that $\bigoplus_i r_i \not\leq_T r$ by adding the needed information to hide the r_i 's.

r is constructed as $\bigcup_i o_i$ where the o_i 's are compatible oracles that represent the left part of r up to column $\mathfrak{b}(i)$

We assume that o_i has been built and that we know $\mathfrak{b}(i)$, and we give the construction for o_{i+1} and $\mathfrak{b}(i+1)$.

The *lost* lemma

Consider all $\varphi_i^\tau(\langle i+1, j \rangle)$ computations for every j and every finite extension τ of o_i .

Look for the first (τ, j) such that it either (i) converges and is $\neq r_{i+1}(j)$, or (ii) for every extension of τ , it diverges.

$\mathfrak{b}(i+1)$ is then taken to be greater than [case (i)] the maximum between $\mathfrak{b}(i)$ and the greatest column reached during the computation, [case (ii)] the greatest column reached in the enumeration of the extensions of o_i ; which is the column from which every extension will make the computation diverge on j .

The *lost* lemma

We need some tool to construct r

LEM $\{r_i : i \in \omega\}$ s.t. $\forall i, r_i <_T r_{i+1}$
 $\exists r$ s.t. $\forall i, r_i \leq_T r$ but $\bigoplus_i r_i \not\leq_T r \leq_T (\bigoplus_i r_i)'$

The *lost* lemma is actually very close to an old classical computability result

5° Republic Argentina. *Publicaciones de la Comisión nacional de la energía atómica*, t. 1, p. 111.

Il signale également un fascicule polycopié : *Contribution du Laboratoire d'Astronomie de Lille*, n° 2. Numéro spécial, à l'occasion du Colloque international de Liège sur *Les particules solides dans les objets astronomiques*.

ARITHMÉTIQUE. — *Sur le semi-réseau constitué par les degrés d'indécidabilité récursive*. Note (*) de M. DANIEL LACOMBE, présentée par M. Émile Borel.

Extension de certains résultats de Kleene et Post (*) et solution de quelques questions posées par ces auteurs (**).

Nous utiliserons dans ce qui suit les définitions et les notations de S. C. Kleene-E. L. Post (*). Nous désignerons par \mathbf{D} l'ensemble des *degrés d'indécidabilité* (ou, pour abrégier : *degrés*) et par \mathbf{D}_λ l'ensemble des degrés arithmétiques ($\mathbf{D}_\lambda \subset \mathbf{D}$). \mathbf{D} et \mathbf{D}_λ sont munis d'une relation d'ordre partiel (notée $<$ et \leq) et d'une opération $a \rightarrow a'$ partout définie (nous désignerons par $a^{(i)}$ le résultat de cette opération itérée i fois à partir de a).

Les deux théorèmes suivants se démontrent au moyen des méthodes classiques (fondées essentiellement sur la *forme normale de Kleene*) complétées par l'utilisation de fonctions majorantes. Les conditions (A) et (B) du théorème I constituent deux cas particuliers d'une condition plus générale que nous ne pouvons énoncer ici, et qu'il serait d'ailleurs intéressant d'élargir.

THÉORÈME I. — Soit $S = u_0, u_1, \dots, u_n, \dots$ une suite infinie de degrés, strictement croissante (c'est-à-dire telle que $i < j$ entraîne $u_i < u_j$) et satisfaisant à l'une ou l'autre des conditions suivantes :

- (A) il existe un degré a tel que, pour tout i , $u_i = a^{(i)}$;
 (B) il existe un degré b tel que, pour tout i , $b < u_i < b'$.

Soit U l'ensemble de degrés défini par la condition :

$$x \in U \Leftrightarrow \text{il existe un } i \text{ tel que } x \leq u_i.$$

Soit d'autre part un degré c n'appartenant pas à U .

Dans ces conditions, on peut énoncer (d'une infinité de façons) de x degrés d_1 et d_2 satisfaisant aux relations suivantes :

(*) Séance du 27 octobre 1954.

(†) *Ann. Math.*, 59, 1954, p. 379-407.

(‡) Ces questions sont en général indiquées, dans l'article cité ci-dessus, par le signe \dagger , lequel renvoie à une Note de la page 380. Les résultats annoncés dans cette Note n'ont pas encore, à notre connaissance, été publiés.

(1) pour tout degré x , on a

$$(x \leq d_1 \text{ et } x \leq d_2) \Leftrightarrow x \in U;$$

(2) c ne vérifie aucune des deux inégalités

$$c \leq d_1 \quad \text{et} \quad c \leq d_2.$$

Remarque 1. — Ce théorème montre que S (ou, ce qui revient au même, U) ne possède pas de borne supérieure précise.

Remarque 2. — La relation (1) montre que le couple (d_1, d_2) ne possède pas de borne inférieure précise. De l'existence de suites S satisfaisant à (A) ou (B) on déduit donc immédiatement que \mathbf{D} ne constitue pas un réseau [résultat démontré par Kleene et Post au moyen d'une suite de type (A)].

Remarque 3. — Kleene et Post ont montré l'existence de suites S satisfaisant à la condition (B), et cela pour n'importe quel degré b . Lorsque, dans cette hypothèse (B), le degré b appartient à \mathbf{D}_λ , il en est de même pour b' et pour tous les u_i (et l'on a $U \subset \mathbf{D}_\lambda$). Cela n'entraîne pas forcément que d_1 et d_2 puissent être pris eux aussi dans \mathbf{D}_λ . La relation (1) montre en effet que l'ensemble U est entièrement déterminé par la donnée de d_1 et d_2 . Or \mathbf{D}_λ est dénombrable (donc aussi l'ensemble des couples formés de deux degrés arithmétiques). Mais Kleene et Post ont montré que les ensembles tels que U , déterminés dans \mathbf{D}_λ par des suites S de \mathbf{D}_λ croissantes et satisfaisant à (B) (avec b dans \mathbf{D}_λ), forment une famille ayant la puissance du continu. Il en résulte que pour certaines de ces suites il n'existe aucun couple (d_1, d_2) formé de degrés arithmétiques et satisfaisant à la relation (1). Le théorème suivant donne une condition suffisante pour l'existence d'un tel couple.

Étant donnée une suite de degrés quelconque $S = u_0, u_1, \dots, u_n, \dots$ et une fonction φ de deux variables (entières ≥ 0), nous dirons que φ énumère S si, pour tout i , la fonction d'une variable φ_i définie par $\varphi_i(x) = \varphi(i, x)$ est de degré u_i .

THÉORÈME II. — Si, dans le théorème I-hypothèse (B), le degré b est arithmétique et si la suite S peut être énumérée par une fonction arithmétique, alors les degrés d_1 et d_2 satisfaisant aux relations (1) et (2) peuvent être pris (d'une infinité de façons) dans \mathbf{D}_λ .

Remarque 1. — Ce résultat reste valable quelque soit le degré c , arithmétique ou non.

Remarque 2. — Si S est énumérable par une fonction arithmétique, il en est de même pour l'ensemble U (qui est toujours dénombrable).

Remarque 3. — Les méthodes de Kleene-Post permettent de déterminer des suites S satisfaisant aux conditions de ce théorème II. Il en résulte que \mathbf{D}_λ ne constitue pas un réseau.

Compt. Rend. Ac. Sci. 239, 1954

The lost lemma

9° Republic Argentina. *Publicaciones de la Comision nacional de la energia atómica, diciembre 1954, n° 1.*
 Il signale également un fascicule polycopié : *Contribution du Laboratoire d'Astronomie de Lille, n° 1*, édité par le Laboratoire national de Liège sur Les

ARITHMÉTIQUE. —
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Daniel Lacombe

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(*) Séance du 27 octobre 1954.

(1) *Ann. Math.*, 59, 1954, p. 379-407.

(2) Ces questions sont en général indiquées, dans l'article cité ci-dessus, par le signe $?$,
 auquel renvoie à une Note de la page 380. Les résultats annoncés dans cette Note n'ont pas
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Ohio State Basketball Ohio State Basketball Ohio State Basketball Ohio State Basketball Ohio State Basketball Ohio State Basketball Ohio State Basketball Ohio State Basketball Ohio State Basketball Ohio State Basketball

Nationwide

ON DEGREES OF RECURSIVE UNSOLVABILITY*

BY CLIFFORD SPECTOR

(Received August 8, 1955)

In Kleene-Post [4]¹ a number of questions concerning the structure of the upper semi-lattice of degrees were left unanswered. The present paper contains the answers to those questions under the scope of [4] Footnote 3. With the exception of the density problem ([4], 2.2), the methods used are variations of those developed in [4]. The construction employed in showing that the degrees are not dense involves a generalization of the methods of [4], and constitutes the main result of this paper (Theorem 4) that there are minimal degrees of recursive unsolvability.² Familiarity with [4] is assumed.³

... of the Mission.
... to Lester Lewis, son of Mr. and Mrs. George Bloom of Great Neck. The wedding was held at the Hotel Pierre, on July 25.

DONOGHUE—HEIDERWEISER—Mr. and Mrs. Louis Gross announce the marriage of their daughter, Mrs. Jeanne G. Heiderweiser, to Edward A. Donoghue, at Holy Kat, in Melbourne, Australia.

HUJSA—KATZ—Mr. and Mrs. Adolph Katz of Woodmere, N. Y., joyfully announce the marriage of their daughter, Carole Joyce, to Charles Jay Hujsa. Ceremony took place at Temple Israel, Lawrence, L. I., on Sun., July 26.

CLIFFORD SPECTOR, MATHEMATICIAN, 30

Special to The New York Times.

PRINCETON, N. J., July 29—Dr. Clifford Spector, Associate Professor of Mathematics at the University of Michigan, died today of a cerebral hemorrhage at Princeton Hospital. He was 30 years old.

Dr. Spector, who had been doing research on a year's leave at the Institute of Defense Analysis here, lived at 327 Walnut Lane.

He previously had taught mathematics at the Ohio State University for five years and was an editor of the Journal of Symbolic Logic, a mathematics periodical.

Dr. Spector graduated from Columbia College, where he was elected to Phi Beta Kappa. He held a Master of Arts degree from Columbia and a doctorate from the University of Wisconsin.

In 1959 he was one of ten United States mathematicians to attend an international conference on symbolic logic at the Warsaw Academy of Science. Last year he did mathematical research at the Institute for Advanced Studies here.

He leaves his wife, the former Lea Eisner; two children, Alan and Judith; his parents, Mr. and Mrs. Robert Spector of New York, and a brother, Gilbert, Professor of Music at Kansas State College.

Anniversaries

GLYNN—Happy Silver Anniversary to my dearest parents, Esther and Leo. With love always, your own, HONEY ANN.

Deaths

ALLEN—Chester A., on July 28, 1961, beloved husband of Katharine M. sister of Chester A., Jr., Betty Allen, Kathleen Kehoe and Alice Stulmer, brother of Conrad Knight and Norman I. Allen, also survived by ten grandchildren, service at the Fairchild Chapel, Franklin Ave., 17th St., Island City, Monday, July 31, 9 A. M. In lieu of flowers kindly send donations to Carver Fund.

HARDY—Marie D., the Larchmont Temple mourns the untimely passing of its member, Marie Hardy, and extends sympathy to her sorrowing family.

MRS. MAURICE MERMEY, President.

HARR—Estima, beloved wife of the late Max Harr, daughter of the late Alexander, loving grandmother and great-grandmother. Services Monday, 2 P. M., "The Riverside," 76th St. and Amsterdam Ave.

HATRY—William A., beloved husband of the late May, devoted father of Patricia and Harry, dear brother of Harry A. and Grace Hatry, wife of tanager, later.

HICKLEY—John J., on July 27, 1961, beloved husband of Mary (nee O'Hara). Funeral from Walter S. Cooke Funeral Home, 215 Westchester Ave., Monday, 9:30 A. M. Solemn Requiem Mass St. Raymond's Church, 4 A. M. Interment Calvary Cemetery.

HOFFMAN—Rose, devoted mother of Ray Fromm, Joan Behrman and Michael, loving grandmother. Services Sun., 11 A. M., "The Riverside," 76th St. and Amsterdam Ave., Monday, July 31, 1961.

HOWARD—Luella, on July 28, 1961, in her 83rd year, beloved wife of the late Herbert G. Howard, daughter of the late Herbert G. and Isabella Buehler, mother of Robert Althoff and William Howard, both of whom are serving in the U. S. Army. Day of funeral services from Walter S. Cooke Funeral Home, 215 Westchester Ave., Monday, 1 P. M., in Woodlawn Cemetery.

JACKSON—Harry, beloved husband of Rebecca, devoted father of Philip, dear brother, loving grandfather. Services Sunday, 12:45 P. M., "The Riverside," 76th St. and Amsterdam Ave.

JACKSON—Henry, The Bronx Chapter of the "The Riverside" records with deep sorrow the passing of Harry Jackson, beloved husband of Buddy Jackson, our esteemed Vice

President. Adolph Celina Levinson and Sylvia Tucker. Services Monday, 9:30 A. M., "The Riverside," 76th St. and Amsterdam Ave.

FRIEND—Cecilia beloved mother of Priscilla Simon, Charlotte, Lea-Fran Rosen and Morris; beloved sister of Albert and Louis "Bole" and Joseph, and loving grandmother, suddenly, on Fire Island, Sat., July 29. Services Mon., July 31, at 1 P. M., "The Riverside," 76th St. and Amsterdam Ave.

GOLDFEEDER—Siman, beloved husband of Leah, devoted father of Elizabeth, Kevin and Eric, and dear brother of Belle Hoke and Irving. Service Sunday, 1 P. M., at the Parc West, 79th St. and Columbus Ave.

GRAHAM—Eric, wife of the late Whidden Graham, mother of W. Conroy Graham. Funeral private. Interment Baltimore, Md.

GRIFFIN—Elizabeth Willett Ryle, on July 28, widow of Henry Arthur Griffin, daughter of the late William and Mary Elizabeth Griffin, mother of Mrs. Aileen Ely, William and Albert Burton. Funeral service at the Chapel of St. James Church, Madison Ave., at 71st St., on Mon., July 31, at 12 noon, in closed service.

HANCOCK—Edith, July 27, formerly of 4754 Richardson Ave., Dr. Furber, 1100 Moss, 162 W. St., Harrison, Newark, N. J.

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Card of Thanks

LEAVITT and Lillian, Dr. Ira D. Leavitt and family gratefully acknowledge the kind and generous expressions of sympathy from their friends and relatives.

The Chapel Of The Four Chaplains, in the spirit of the selfless men who act as its inspiration, is dedicated to serving people of all faiths. Every way in which we serve is aided by the most modern methods available; combined with the old-fashioned virtues of courtesy and dignity.

When you choose Universal, you select men and women who are sincerely interested in you and who are prepared to give you their very best efforts. This is what you have a right to expect. This is what you get at Universal.

Universal
FUNERAL CHAPEL OF THE

The Lost Lemma

In 1925 he put the folk song "Home on the Range" into sheet-music form.

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to Lester Levin, son of Mr. and Mrs. George Bland of Great Neck. The wedding was held at the Hotel Pierre, on July 29, 1961.

WEINWEISER—Mr. and Mrs. William E. Weinwaiser, of 320 West 93rd St., New York City, announce the marriage of their daughter, Jeanne G. Weinwaiser, to Bernard A. Gotschke, on July 19, in Melbourne, Australia.

HUISA-KATZ—Mr. and Mrs. Maloh Katz of Westchester, N. Y., joyfully announce the marriage of their daughter, Carole Joyce, to Charles Jay Huisa. Ceremony took place at Temple Israel, Lawrence, L. I., on Sun., July 29, 1961.

SEGUL-DELL'ARIA—Dr. and Mrs. Salvatore Dell'Arvia of 103 E. 84th St. announce the marriage of their daughter, Marina, to Charles J. Segul, at the St. Ignatius Loyola R. C. Church, Thruway, July 27, 1961.

SHAPIRO-SCHAPIRO—Mr. and Mrs. Morris Shapiro, Lake Success, N. Y., announce the marriage of their daughter, Marcia Eileen, to Dr. Herman Shapiro, son of Mr. and Mrs. Gershon Shapiro, Bayshore, N. Y.

Anniversaries

GLYNN—Have Silver Anniversary to my dearest parents, Esther and Leo. With love toward your own, HONEY ANN.

Deaths

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ALLAN—Chester A., the Board of Trustees of the John Hope Foundation records with profound sorrow the death of its trustee, Chester A. Allen.

ALLEN—Margaret, nee Lunn, Howard A. Lee, on July 28, 1961, at 1800 W. 11th St., Los Angeles, California, aged 70 years, service at the Hollywood Memorial Chapel, 1810 W. 11th St., Los Angeles, California, on Monday, August 6, 1961, at 2 P. M., interment at Hollywood Memorial Cemetery.

ANUSCIC—Alaister, nee Terence M. Hee, burton A. M. O'Leary, on July 28, 1961, at 1800 W. 11th St., Los Angeles, California, aged 70 years, service at the Hollywood Memorial Chapel, 1810 W. 11th St., Los Angeles, California, on Monday, August 6, 1961, at 2 P. M., interment at Hollywood Memorial Cemetery.

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BRODERICK—John, nee Julia, on July 28, 1961, at 1800 W. 11th St., Los Angeles, California, aged 70 years, service at the Hollywood Memorial Chapel, 1810 W. 11th St., Los Angeles, California, on Monday, August 6, 1961, at 2 P. M., interment at Hollywood Memorial Cemetery.

FRIEND—Cecilia, beloved mother of Priscilla Simon, Charlotte, Lea-Fran Rosen and Morris; beloved sister of Albert and Louis; beloved grandmother of many; beloved and loving grandmother, suddenly on Fire Island, Sat. July 29, Services Mon., July 31, at 1 P. M. at "The Riverside," 76th St. and Broadway Ave.

GOLDFEEDER—Simón, beloved husband of Leah, devoted father of Elizabeth, Kevin Yanni, Evelyn, Edward, brother of Belle Hope and Irving; Service Sunday, 1 P. M., at the Parc West, 79th St. and Columbus Ave.

GRAHAM—Erica O'Brien, wife of the late Whitford Graham, mother of W. Conroy Graham. Funeral private. Interment Baltimore, Md.

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HANCOCK—Edith, July 27, formerly of 4754 Michigan Ave., dr. Furber, 1000 Massachusetts St., Hingham, Massachusetts. Funeral service at the Holy Trinity R. C. Church, 1000 Massachusetts St., Hingham, Massachusetts, on Monday, July 31, at 10 A. M.

HARDT—Marie D. (nee Kinkels), on July 29, 1961, of 47 Woodbine Ave., Larchmont, N. Y., beloved wife of John P., loving mother of Arthur, Mark, and Louise; grandmother of Paul; Residing at the Fox Funeral Home, 80 Post Rd., Larchmont, N. Y. Interment at the Holy Trinity R. C. Church, 1000 Massachusetts St., Hingham, Massachusetts, on Monday, July 31, at 10 A. M.

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HARRY—William A., beloved husband of the late Mrs. Gertrude L. of Patricia and Josephine; devoted father of Herbert G. and F. Russell Howard, sister of Alice Bauer, grandmother of Barbara Alshab and Arthur H. Hagan. Mrs. Howard was active for many years in the Bronx Day Nursery, Services at the Walter B. Cooke Funeral Home, 100 W. 110th St., Mon., 1 P. M., Interment Woodlawn Cemetery.

JACKSON—Harry, beloved husband of Rebecca, devoted father of Philip, dear brother, loving grandfather. Services Sunday, 12:45 P. M., "The Riverside," 76th St. and Amsterdam Ave.

JACKSON—Harry, the Bronx Chapter of the B'nai B'rith records with deep sorrow the passing of Harry Jackson, beloved husband of Buddy Jackson, our esteemed Vice President, on July 28, 1961, at 1800 W. 11th St., Los Angeles, California, aged 70 years, service at the Hollywood Memorial Chapel, 1810 W. 11th St., Los Angeles, California, on Monday, August 6, 1961, at 2 P. M., interment at Hollywood Memorial Cemetery.

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KAUFMAN—Mortimer, nee Sarah, on July 28, 1961, at 1800 W. 11th St., Los Angeles, California, aged 70 years, service at the Hollywood Memorial Chapel, 1810 W. 11th St., Los Angeles, California, on Monday, August 6, 1961, at 2 P. M., interment at Hollywood Memorial Cemetery.

KEECHAN—Elizabeth S., of 30 Irvington Ave., Elizabeth, N. J., former of Newark, wife of Dr. Charles K. Keechan, and Elizabeth M. and Mrs. John E. Keechan; survived by her son, Dr. John E. Keechan, 414 Westminster Ave., Elizabeth, N. J., on Sat. July 29, at 9:30 A. M. B'nai B'rith Home, 330 Coney Island Ave., Brooklyn.

OFFENTHER—Isidor, devoted husband of Frances, devoted father of Harold, darling grandfather, services today, 10:30 A. M., at Heiman's Funeral Chptrs., 1342 Grand Concourse, Bronx.

OLCAY—Astrid Van Sandwoud, July 28. Survived by his wife, four sons, a daughter and six grandchildren. Services Monday, July 31, 11 A. M., Uccitt residence, West Squirrel, Maine. Donations may be made to St. Andrews Hospital Fund, Booth Bay Harbor.

PALMER—Minnie V., beloved mother of Arthur M. Lovins, grandmother of Leslie Frank, buried sister of Gertrude R. Katz, Selma Lavin and Samuel S. Shapiro and devoted daughter. Services Monday, 2 P. M., "The Riverside," 76th St. and Amsterdam Ave., Brooklyn.

PESNER—Minnie, beloved mother of Jacob, Lily Katz and Herman, dear grandmother and great-grandmother. Services today, 10 A. M., Garlick's Parkside, 1700 Coney Island Ave. (Ave. N), Brooklyn.

PLUNKER—Jack, beloved husband of Sophie, devoted father of David and Sidney Platt, darling grandfather and dear brother. Interceded Fri., July 28, 1961.

PRASKHER—Jack L. We sorrowfully announce the passing of our member West 300 Institutional Synagogue, LESLIE GUELL, prominent.

RADA—Rudi, nationally known photographer, beloved husband of Annette and loving son of Jeannette Kala, died suddenly, July 23, at Saint Michael's.

RATHENBERG—Andree, of 54 Ocean Boulevard, Atlantic Highlands, on July 27, 1961, wife of August Rathenberg, mother of John and Richard, daughter of Mrs. Andree Keechan. A High Mass of Requiem will be held at 10:30 A. M., on Monday, August 6, 1961, at the Holy Trinity R. C. Church, 1000 Massachusetts St., Hingham, Massachusetts.

RECKLIN—Dr. Abramian Recklin, his successful son-in-law, Dr. Recklin was a devoted member of the Pediatric Staff of this hospital for many years.

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The Chapel Of The Four Chaplains

in the spirit of the selfless men who act as its inspiration, is dedicated to serving people of all faiths. Every way in which we serve is aided by the most modern methods available; combined with the old-fashioned virtues of courtesy and dignity.

When you choose Universal, you select men and women who are sincerely interested in you and who are prepared to give you their very best efforts. This is what you have a right to expect. This is what you get at Universal.



The Lost Lemma

In 1925 he put the folk song "Home on the Range," into sheet-music form.

CLIFFORD SPECTOR, MATHEMATICIAN, 30

Special to The New York Times.

PRINCETON, N. J., July 29.—Dr. Clifford Spector, Associate Professor of Mathematics at the University of Michigan, died today of a cerebral hemorrhage at Princeton Hospital. He was 30 years old.

Dr. Spector, who had been doing research on a year's leave at the Institute of Defense Analysis here, lived at 327 Walnut Lane.

He previously had taught mathematics at the Ohio State University for five years and was an editor of the Journal of Symbolic Logic, a mathematics periodical.

Dr. Spector graduated from Columbia College, where he was elected to Phi Beta Kappa. He held a Master of Arts degree from Columbia and a doctorate from the University of Wisconsin.

In 1959 he was one of ten United States mathematicians to attend an international conference on symbolic logic at the Warsaw Academy of Science. Last year he did mathematical research at the Institute for Advanced Studies here.

He leaves his wife, the former Lea Eisner; two children, Alan and Judith; his parents, Mr. and Mrs. Robert Spector of New York, and a brother, Gilbert, Professor of Music at Kansas State College.

to Lester Levin, son of Mr. and Mrs. George Bloom of Great Neck. The wedding was held at the Hotel Pierre, on July 25, 1961.

WEINWEISER—Mr. and Mrs. Morris announce the marriage of their daughter, Bernice G. Weinweiser, to Edward A. Conroy, on July 19, in Melbourne, Australia.

MUJSA-KATZ—Mr. and Mrs. Malach Katz of Woodmere, L. I., joyfully announce the marriage of their daughter, Carole Joyce, to Charles Jay Mujsa. Ceremony took place at Temple Israel, Lawrence, L. I., on Sun., July 23, 1961.

SEGUL-DELL'ARIA—Dr. and Mrs. Salvatore Dell'Arvia of 103 E. 84th St. announce the marriage of their daughter, Marina, to Charles J. Segul, at the St. Ignace Loyola R. C. Church, Thruway, July 27, 1961.

SHAPIRO-SHAPIRO—Mr. and Mrs. Morris Shapiro, Lake Success, N. Y., announce the marriage of their daughter, Marcia Eileen, to Dr. Herman Shapiro, son of Mr. and Mrs. Gershon Shapiro, Bayville, N. Y.

Anniversaries

GLYNN—Happy Silver Anniversary to my dearest parents, Esther and Leo. With love always, your own, HONEY ANN.

Deaths

ALLEN—Chester A., on July 28, 1961, beloved husband of Katharine M. (nee) Allen, Jr., Betty Allen, Kathleen Kathleen and Alice Stupiner, sister of Conrad Knight and Norman I. Allen, also survived by ten grandchildren, service at the Fairchild Chapel, Franklin Ave., 127th St., Island City, Monday, at 9 P. M. Invitations to your Church Chesty would be appreciated.

ALLEN—Chester A., the Board of Trustees of the Pain Home Foundation records with profound sorrow the death of its trustee, Chester A. Allen.

ALLEN—MARTIN—Lynn, Howard A., Lee, Lorraine, and Louis, all of which, O. L. ANUSCIC—Albin, Terrence M., HEBURTON, A. M., OUL, A. M., ANUSCIC—Washington Ave. E. Funeral Rigdon.

AQUA—Al, July 26, dear father Francis.

BADINI-LLI—Julia, on July 25, 1961, of Luccombe, N. Y., beloved wife of the late John, dear mother of Irene Fiorillo and John, Jr. Burial at the Westchester Funeral Home, Inc., Algonquin Ave. at Main St., Luccombe. Requiem Mass in the church.

BRODERICK—Mrs. M. M. Broderick, 76th St. and Amsterdam Ave. Funeral services Sunday, 12:45 P. M., "The Riverside," 76th St. and Amsterdam Ave.

JACKSON—Mary, The Bronx Chapter of the B'nai B'rith records with deep sorrow the passing of Harry Jackson, beloved husband of Buddy Jackson, our esteemed Vice President.

Funeral services Sunday, 12:45 P. M., "The Riverside," 76th St. and Amsterdam Ave.

Adolph, Celia Levin and Sylvia Broderick. Services Monday, 9:30 A. M., "The Riverside," 76th St. and Amsterdam Ave.

FRIEND—Cecilia, beloved mother of Priscilla Simon, Charlotte, Lea-Fran Rosen and Morris; beloved sister of Albert and Louis; wife of the late Harry Friend, loving grandmother, suddenly on Fire Island, Sat. July 29. Services Mon., July 31, at 3 P. M., at "The Riverside," 76th St. and Amsterdam Ave.

GOLDFEEDER—Simma, beloved husband of Leah, devoted father of Elizabeth, Kevin and Eric, dear brother of Belle Hope and Irving. Service Sunday, 1 P. M., at the Parc West, 79th St. and Columbus Ave.

GRAHAM—Bertha O'Brien, wife of the late Whitford Graham, mother of W. Conroy Graham. Funeral private. Interment Baltimore, Md.

GRIFFIN—Elizabeth Willett Ryle, on July 28, widow of Henry Arthur Griffin, daughter of the late William and Mary Elizabeth Ryle, mother of Mrs. Aired Joy, William and Albert Burton. Funeral service at the Chapel of St. James Church, Madison Ave. at 71st St., on Mon., July 31, at 12 noon.

HANCOCK—Ellis, July 27, formerly of 4754 Michigan Ave., dr. Funeral Home, 835 W. 111th St., Hershong, Cleveland, Fri., July 29, 1961, of 47 Woodbine Ave., Larchmont, N. Y., beloved wife of John P., loving mother of Esther Marks, and Louise Blümann, grandmother of Paul Reussing at the Fox Funeral Home, 60 Post Rd., Larchmont, on Monday, 10 A. M. Religious service at the Larchmont Temple, Monday, 11 A. M. In lieu of flowers kindly send donations to Cancer Fund.

HARDY—Marie D., the Larchmont Temple mourns the untimely passing of its member, Marie Hardy, and extends sympathy to her sorrowing family.

MRS. MAURICE REMEY, President, **HART**—Estima, beloved wife of the late Louis Hart, dear mother of John M., Benjamin and the Alexander, loving grandmother and great-grandmother. Services Monday, 1 P. M., "The Riverside," 76th St. and Amsterdam Ave.

HATRY—William A., beloved husband of the late Mary, devoted father of Patricia and

Alicia and Martin. Services Monday, 1 P. M., "The Riverside," 76th St. and Amsterdam Ave.

O'BRIEN—Elizabeth S., of 30 Irvington Ave., Elizabeth, N. J., former of Newark, wife of Joseph J. O'Brien and mother of Elizabeth M. and Cecy John E. O'Brien, funeral from the Jews J. Higgins and Son Mortuary, 414 Westminster Ave., Elizabeth, on Tues. Aug. 1, at 9:30 A. M. St. Bonaventura Mass. Immaculate Conception Church, 10 A. M.

OFFERTHER—Isidor, beloved husband of Fannie, devoted father of Harold, darling grandfather, services today, 10:30 A. M., at Heiman's Funeral Chptrs., 1242 Grand Concourse, Bronx.

OLCAY—Aired Van Sandover, July 28. Survived by his wife, two sons, a daughter and six grandchildren. Services Monday, July 31, 11 A. M., Uccit residence, West Spaulding, Maine. Burial services may be made to St. Andrews Hospital Fund, Booth Bay Harbor.

PALMER—Minnie V., beloved mother of Arthur M. Loving, grandmother of Leslie Frank, buried sister of Gertrude R. Katz, Selma Lavin and Samuel S. Shapiro and devoted daughter. Services Monday, 2 P. M., "The Riverside," 310 Corey Island Ave., Brooklyn.

PESNER—Minnie, beloved mother of Jacob, Lily Katz and Herman, dear grandmother and great-grandmother. Services today, 10 A. M., Gurlick's Parkside, 1700 Wood Island Ave. (Ave. N), Brooklyn.

PLUNKOFF—Jack, beloved husband of Sophie, devoted father of David and Sidney Platt, darling grandfather and dear brother. Services Fri., July 28, 1961.

PRAKHER—Jack L. We sorrowfully announce the passing of our member, West 340 Institutional Synagogue, LESLIE QUELLEN, President.

RADA—Rudi, nationally known photographer, beloved husband of Annette and loving son of Jeannette Rada, died suddenly, July 23, 1961, in his home, Ft. Lee, Va.

RATHER—Andree, of 54 Ocean Boulevard, Atlantic Highlands, on July 27, 1961, wife of August Rathemacher, mother of John and Richard, daughter of Mrs. Andree Keeschen. A High Mass of Requiem will be

celebrated at the Holy Trinity Church, Atlantic Highlands, on Monday, August 1, at 10:30 A. M.

ROSEN—Mortimer, beloved husband of the late Sarah, devoted father of Benjamin and Joseph, loving grandfather. Services Monday, 10:30 A. M., at the Holy Trinity Church, Atlantic Highlands, on Monday, August 1, at 10:30 A. M.

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Card of Thanks
LEAVITT—Lillian, Dr. Isaac M. Leavitt and family graciously acknowledge the kind and gracious expression of sympathy from their friends and relatives.

The Chapel Of The Four Chaplains, in the spirit of the selfless men who act as its inspiration, is dedicated to serving people of all faiths. Every way in which we serve is aided by the most modern methods available; combined with the old-fashioned virtues of courtesy and dignity.

When you choose Universal, you select men and women who are sincerely interested in you and who are prepared to give you their very best efforts. This is what you have a right to expect. This is what you get at Universal.

universal FUNERAL CHAPEL OF THE

a and b form an exact pair for a degree set C if both are above all degrees in C any degree below both is also below some degree in C

Every countable set of degrees in which every pair of elements is bounded has an exact pair

DEF THM

The recursively inaccessible case

ι_0 : ω -sequence β_i of admissibles

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β_i codable by r_i in L_{β_i+1}

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If the codes appear soon enough, same case than ι_0

The recursively inaccessible case

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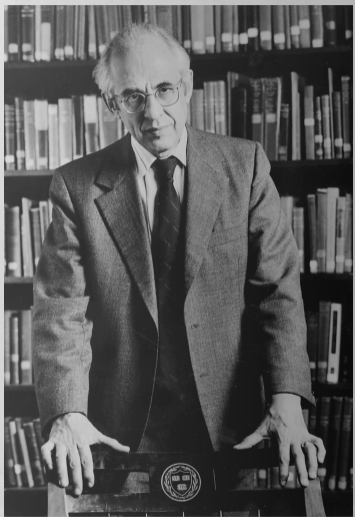
If the codes appear soon enough, same case than ι_0

Where are the codes for the sequence members?



Harvard University

Hilary Whitehall Putnam



Putnam/definability gaps

THM

There are arbitrarily long gaps in L where no news reals appear

Putnam gaps

Putnam/definability gaps

THM There are arbitrarily long gaps in L where no new reals appear Putnam gaps

Let $\beta > \alpha$ be countable ordinals such that there is an elementary embedding $j : L_\beta \rightarrow L_{\omega_2}$ with *critical point* $cr(j) \geq \alpha$.

For every $\gamma < cr(j)$,

$L_{\omega_2} \models$ “No new reals appear between ranks ω_1 and $\omega_1 + \gamma$.”

No new reals thus appear between $cr(j)$ and $cr(j) + \gamma$, by elementarity and absoluteness.

Putnam/definability gaps

THEM

There are arbitrarily long gaps in L where no new reals appear

Putnam gaps

LEM

If new reals appear at $\alpha + 1$, then among them is an arithmetical copy E_α of L_α

E_α is an arithmetical copy of L_α if there is one-one function f from L_α to ω (and onto the field of E_α) such that $\forall x, y \in L_\alpha$,
 $x < y \iff \langle f(x), f(y) \rangle \in E_\alpha$

Putnam/definability gaps

THEM There are arbitrarily long gaps in L where no new reals appear

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Later seen as Jensen's mastercodes

Putnam/definability gaps

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Later seen as Jensen's mastercodes

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$\mathcal{J}_0 = \emptyset$, $\mathcal{J}_{\omega \cdot \xi} = L_\xi$, $\mathcal{J}_{\omega \cdot \xi + n} = \Delta_n(L_\xi)$, where λ is a limit ordinal and $\Delta_n(X)$ is the set of all subsets of X definable with parameters in $\langle X, \in \rangle$ by both Σ_n and Π_n first order formulae.

Putnam/definability gaps

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There exists α such that $L_\alpha \prec L_{\omega_1}$, α is thus not definable in L_{ω_1} .

There is a countable $\nu > \alpha$ such that $L_\nu \prec L_{\omega_1}$, and α is already not definable in L_ν .

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σκέδιος

The ordinal ν_0 , which is the least ν such that there is an ordinal α not definable in L_ν , can be characterized as the least η such that there exists an ordinal $\delta < \eta$ such that $L_\delta \prec L_\eta$.

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$\Delta_\lambda(L)$, where λ is a limit ordinal and $\Delta_\nu(X)$ is the set of all subsets of X definable with parameters in $\langle X, \in \rangle$ by both Σ_n and Π_n first-order formulae.

Putnam/definability gaps

ν_0 is clearly \leq the least such η , η_0 , since whenever one has $L_\alpha \prec L_\beta$, α is not definable in L_β .

Now, suppose that $\nu_0 < \eta_0$, in other words, for all $\delta < \nu_0$, $L_\delta \not\prec L_{\nu_0}$. By Löwenheim-Skolem there is a countable elementary submodel of L_{ν_0} . Take the \subseteq -least such model M . By the Condensation Lemma, there is an $\alpha < \nu_0$ and an isomorphism j such that the Mostowski collapse of M is isomorphic to L_α via j . j cannot be trivial as this would mean that $L_\alpha \prec L_\delta$, although $\delta < \nu_0$ and ν_0 is the least such ordinal. We can thus consider κ , the critical point of j . Since $L_\alpha \cong M \prec L_{\nu_0}$, $L_\kappa \prec L_{j(\kappa)}$. But then κ cannot be definable in $L_{j(\kappa)}$, and thus $\nu_0 \leq j(\kappa)$. But $j(\kappa) < \nu_0$, contradiction.

We can characterize the least such definability gap

Definable/codable/countable

Def

- α *definable* at γ if definable without parameters in L_γ
- α *codable* at γ if appears in $L_{\gamma+1}$ a real coding α
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$\alpha < \omega_1^L$ is codable at some level
if α is countable at β ,
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α countable at γ if $L_\gamma \models \text{“}\alpha \text{ is countable”}$.

Consider $\kappa = \aleph_\alpha$. κ is definable as the greatest cardinal in L_{κ^+} .

(Here κ^+ denotes the least ordinal of cardinality greater than κ .)

And thus α is also definable in L_{κ^+} .

Löwenheim-Skolem's theorem, in conjunction with Mostowski's lemma and the Condensation Lemma, provides the countable β such that α is definable in L_β .

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There exists an ordinal $\alpha < \omega_1^L$ which is definable at a β ,
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$\alpha < \omega_1^*$ is codable at some level

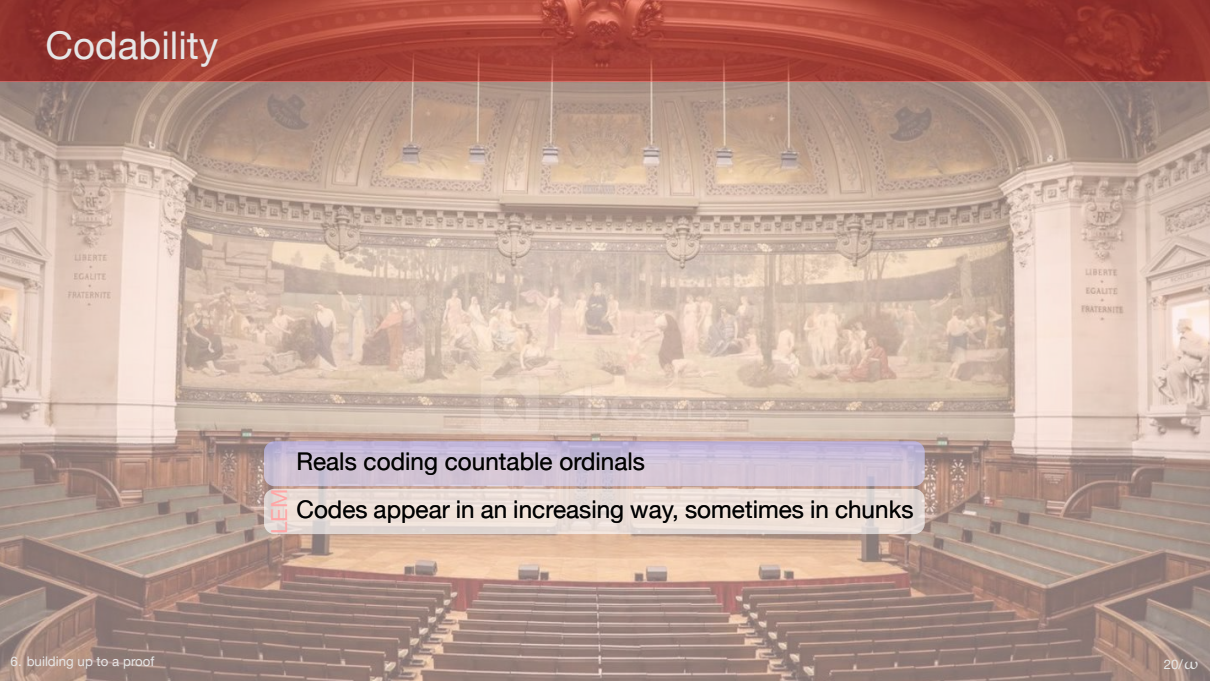
Memorable ordinals: ordinals α for which there exists β such that for any countable $\gamma \geq \beta$, α is still definable at γ .

Any countable τ such that $L_\tau \prec L_{\omega_1}$ is such an upper bound: if α is definable at β , take δ above τ and β such that $L_\delta \prec L_{\omega_1}$. We then have $L_\tau \prec L_\delta \prec L_{\omega_1}$. α is thus definable at δ , since δ is above β , and also at τ . τ is therefore above α and any other definable ordinal. In fact, the least non-memorable ordinal τ_0 is the least ordinal τ with uncountably many elementary extensions $L_\tau \prec L_\gamma$.

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Reals coding countable ordinals



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Reals coding countable ordinals

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If α and β are in the same codability gap and $\alpha \ll \beta$
then $r_\alpha <_T r_\beta$

The case of codability gaps

Successor admissible case

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$$\omega_1^{\text{CK}, r_\beta} \geq \alpha \text{ but } r_\alpha \not\leq_T r_\beta, \text{ so } \omega_1^{\text{CK}, r_\beta} = \alpha$$

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Recursively inaccessible case

Everything happens in L_γ
where the first code for α appears

7. sacks' and jensen's theorems revisited

Est-il possible de raisonner sur des objets qui ne peuvent être définis en un nombre fini de mots ? Est-il possible même d'en parler en sachant de quoi l'on parle, et en prononçant autre chose que des paroles vides ? Ou au contraire doit-on les regarder comme impensables ? Quant à moi je n'hésite pas à répondre que ce sont de purs néants.

—Poincaré, *La logique de l'infini*, 1909

How far does it work?

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Optimality results concerning the degree of r



Universität Bonn

Ronald B. Jensen



Jensen's generalization

Sacks' theorem but for a sequence of admissibles?

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But we have to be careful with the hypothesis

Jensen's generalization

σχέδιο

sequence of α admissibles
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Use of infinite time Turing machines in generalized lemma

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$\beta + 1$ -th in the sequence should not be decodable from r in L_{τ_β}

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THE MATHS